Linking KNP Instructional Resources to Add+VantageMR®

Add+VantageMR® is a professional development and set of assessment interviews created by the US Math Recovery Council. For more information about this training, please go to https://www.mathrecovery.org. Through AVMR, teachers learn numeracy progressions as well as assessment tools for determining a student's progress along these progressions.

The activities in the KNP bank of Instructional Resources are aligned to the AVMR progressions in all strands except Fractions. The prefix indicates the targeted strand and the suffix indicates the targeted level. For example, activity Nb 1109.5 targets backward counting and is designed to move a student from AVMR level 4 to level 5 in backward counting.

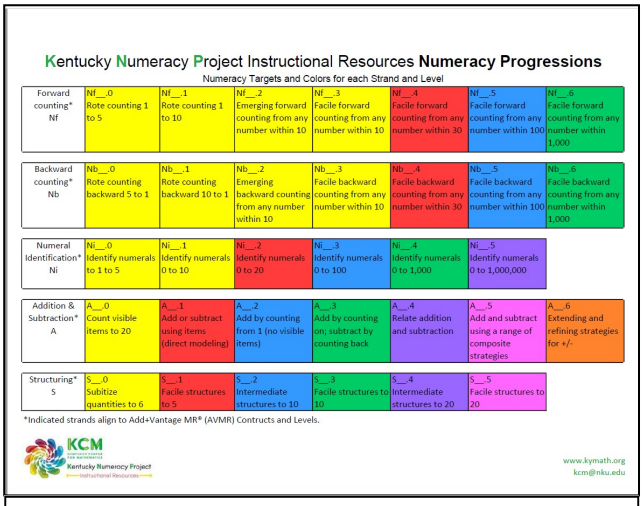
Numeracy Strand

The prefix indicates the numeracy strand:

Number Words and Numerals
- **[Nf]** – Number words forward
- **[Nb]** – Number words backward
- **[Ni]** – Numeral ID

[A] Addition and Subtraction
[S] Structuring within 5, 10 or 20
[M] Multiplication and Division
[T] Base Ten Arithmetical Strategies
[F] Fractions (not aligned to AVMR)

Each strand and level has been assigned a color which can be seen in our Numeracy Progressions Chart. The color system was developed by teachers to organize activities and instructional materials. The colors are used in the KNP on the student instructions page for easy identification by both teacher and student. Some teachers additionally use colored stickers, colored folders, and/or labeled bins to make identifying the color of an activity easier.



Kentucky Numeracy Project Instructional Resources **Numeracy Progressions**

Numeracy Targets and Colors for each Strand and Level

Forward counting* Nf	Nf__0 Rote counting 1 to 5	Nf__1 Rote counting 1 to 10	Nf__2 Emerging forward counting from any number within 10	Nf__3 Facile forward counting from any number within 10	Nf__4 Facile forward counting from any number within 30	Nf__5 Facile forward counting from any number within 100	Nf__6 Facile forward counting from any number within 1,000
Backward counting* Nb	Nb__0 Rote counting backward 5 to 1	Nb__1 Rote counting backward 10 to 1	Nb__2 Emerging backward counting from any number within 10	Nb__3 Facile backward counting from any number within 10	Nb__4 Facile backward counting from any number within 30	Nb__5 Facile backward counting from any number within 100	Nb__6 Facile backward counting from any number within 1,000
Numeral Identification* Ni	Ni__0 Identify numerals to 1 to 5	Ni__1 Identify numerals 0 to 10	Ni__2 Identify numerals 0 to 20	Ni__3 Identify numerals 0 to 100	Ni__4 Identify numerals 0 to 1,000	Ni__5 Identify numerals 0 to 1,000,000	
Addition & Subtraction* A	A__0 Count visible items to 20	A__1 Add or subtract using items (direct modeling)	A__2 Add by counting from 1 (no visible items)	A__3 Add by counting on; subtract by counting back	A__4 Relate addition and subtraction	A__5 Add and subtract using a range of composite strategies	A__6 Extending and refining strategies for +/-
Structuring* S	S__0 Subitize quantities to 6	S__1 Facile structures to 5	S__2 Intermediate structures to 10	S__3 Facile structures to 10	S__4 Intermediate structures to 20	S__5 Facile structures to 20	

*Indicated strands align to Add+Vantage MR® (AVMR) Contructs and Levels.

KCM Kentucky Numeracy Project

www.kymath.org
kcm@nku.edu

Numeracy Progressions Chart

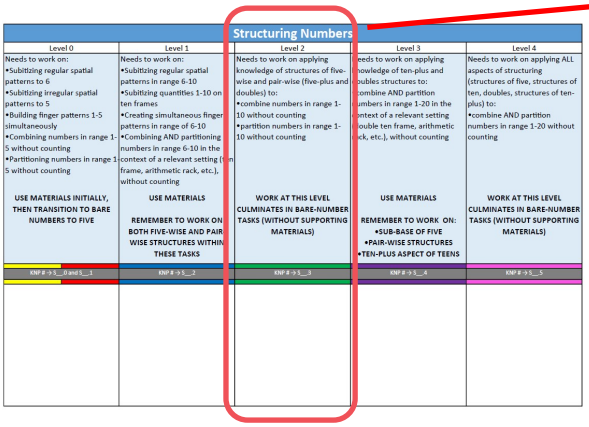
For example, all Nb level 4 activities are designated red and target backward counting within 30. Nb level 5 activities are designated blue and target backward counting within 100. Many teachers find that the color system lets them easily distinguish between these activities, using the appropriate one for a specific student or group.

Students can also use the color system to track progress and select activities. For example, a teacher might have their Nb activities in a tub. Each student would be assigned a color depending on their current progress and would be given the freedom to choose any Nb activity in their designated color. It's important to note that a color in one strand does NOT indicate a student will be the same color in a different strand. Teachers with AVMR training will have the tools and knowledge to determine a student's point in each progression.

Resources for linking KNP and AVMR

AVMR teachers may find the following resources useful when using the KNP.

- Differentiation worksheets are organizers that link AVMR Constructs and Levels to KNP resources.
- Tub labels can be used for organizing materials.



Structuring Numbers				
Level 0	Level 1	Level 2	Level 3	Level 4
Needs to work on: •Subitizing regular spatial patterns to 6 •Subitizing irregular spatial patterns to 5 •Building finger patterns 1-5 simultaneously •Combining numbers in range 1-5 without counting •Partitioning numbers in range 1-5 without counting	Needs to work on: •Subitizing regular spatial patterns in range 6-10 •Subitizing quantities 1-10 on ten frames •Creating simultaneous finger patterns in range of 6-10 •Combining AND partitioning numbers in range 6-10 in the context of a relevant setting (ten frame, arithmetic rack, etc.), without counting	Needs to work on applying knowledge of structures of five-wise and pair-wise (five-plus and doubles) to: •combine numbers in range 1-10 without counting •partition numbers in range 1-10 without counting	Needs to work on applying knowledge of ten-plus and doubles structures to: •combine AND partition numbers in range 1-20 in the context of a relevant setting (double ten frame, arithmetic rack, etc.), without counting	Needs to work on applying ALL aspects of structuring (structures of five, structures of ten, doubles, structures of ten-plus) to: •combine AND partition numbers in range 1-20 without counting
USE MATERIALS INITIALLY, THEN TRANSITION TO BARE NUMBERS TO FIVE	USE MATERIALS REMEMBER TO WORK ON BOTH FIVE-WISE AND PAIR WISE STRUCTURES WITHIN THESE TASKS	WORK AT THIS LEVEL CULMINATES IN BARE-NUMBER TASKS (WITHOUT SUPPORTING MATERIALS)	USE MATERIALS REMEMBER TO WORK ON: •SUB-BASE OF FIVE •PAIR-WISE STRUCTURES •TEN-PLUS ASPECT OF TEENS	WORK AT THIS LEVEL CULMINATES IN BARE-NUMBER TASKS (WITHOUT SUPPORTING MATERIALS)
KNP # → S__.1 and S__.2	KNP # → S__.2	KNP # → S__.3	KNP # → S__.4	KNP # → S__.5

Example of a Differentiation worksheet

Reminder of key components of this level in the AVMR progression

Indicates appropriate KNP activities. In this case, we recommend Structuring level 3 activities to move students FROM level 2 TOWARD Level 3

Space to either list students or make notes about instructional activities.

Structuring Numbers

Level 2

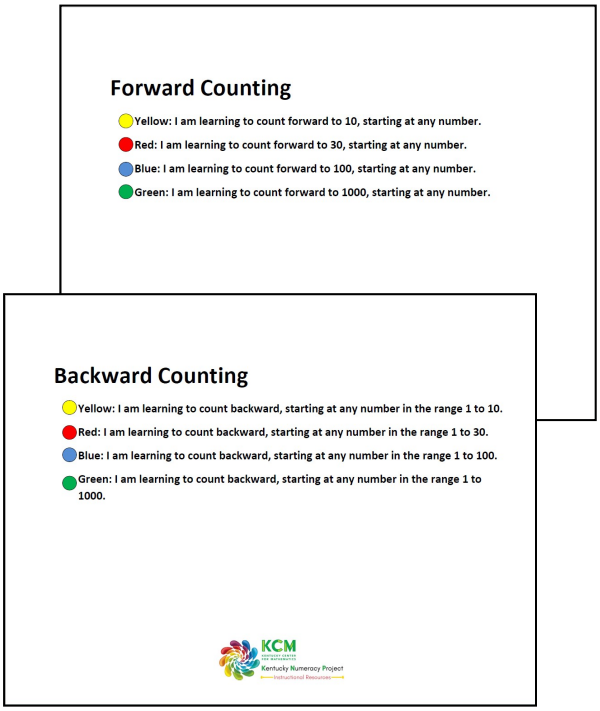
Needs to work on applying knowledge of structures of five-wise and pair-wise (five-plus and doubles) to:

- combine numbers in range 1-10 without counting
- partition numbers in range 1-10 without counting

WORK AT THIS LEVEL CULMINATES IN BARE-NUMBER TASKS (WITHOUT SUPPORTING MATERIALS)

KNP # → S__.3

Tub labels



Kentucky Numeracy Project Instructional Resources **Numeracy Progressions**

Numeracy Targets and Colors for each Strand and Level

Forward counting* Nf	Nf__.0 Rote counting 1 to 5	Nf__.1 Rote counting 1 to 10	Nf__.2 Emerging forward counting from any number within 10	Nf__.3 Facile forward counting from any number within 10	Nf__.4 Facile forward counting from any number within 30	Nf__.5 Facile forward counting from any number within 100	Nf__.6 Facile forward counting from any number within 1,000
Backward counting* Nb	Nb__.0 Rote counting backward 5 to 1	Nb__.1 Rote counting backward 10 to 1	Nb__.2 Emerging backward counting from any number within 10	Nb__.3 Facile backward counting from any number within 10	Nb__.4 Facile backward counting from any number within 30	Nb__.5 Facile backward counting from any number within 100	Nb__.6 Facile backward counting from any number within 1,000
Numeral Identification* Ni	Ni__.0 Identify numerals to 1 to 5	Ni__.1 Identify numerals 0 to 10	Ni__.2 Identify numerals 0 to 20	Ni__.3 Identify numerals 0 to 100	Ni__.4 Identify numerals 0 to 1,000	Ni__.5 Identify numerals 0 to 1,000,000	
Addition & Subtraction* A	A__.0 Count visible items to 20	A__.1 Add or subtract using items (direct modeling)	A__.2 Add by counting from 1 (no visible items)	A__.3 Add by counting on; subtract by counting back	A__.4 Relate addition and subtraction	A__.5 Add and subtract using a range of composite strategies	A__.6 Extending and refining strategies for +/-
Structuring* S	S__.0 Subitize quantities to 6	S__.1 Facile structures to 5	S__.2 Intermediate structures to 10	S__.3 Facile structures to 10	S__.4 Intermediate structures to 20	S__.5 Facile structures to 20	

*Indicated strands align to Add+Vantage MR® (AVMR) Contructs and Levels.

KCM Kentucky Center for Mathematics
Kentucky Numeracy Project
Instructional Resources

www.kymath.org
kcm@nku.edu

Kentucky Numeracy Project Instructional Resources **Numeracy Progressions**

Numeracy Targets and Colors for each Strand and Level

Base Ten Arithmetical Strategies* T	T__.0 Emerging understanding that 2-digit numbers are composed of tens and ones	T__.1 Solve 2-digit +/- with materials by counting by 10s OR by 1s	T__.2 Solve 2-digit +/- with materials using strategies based on place value	T__.3 Beginning to solve 2-digit +/- without materials using strategies based on place value	T__.4 Solve 2-digit +/- without materials using a range of strategies	T__.5 Solve 3-digit +/- without materials using a variety of strategies	T__.6 Extending and refining efficient strategies for multi-digit +/-
Multiplication and Division* M	M__.0 No activities at the level	M__.1 Build and share items into equal groups	M__.2 Count equal groups using stress or skip counting	M__.3 Count items arranged in equal groups with only group markers visible (items within groups are not visible)	M__.4 Multiply and divide within 100 using counting strategies	M__.5 Multiply and divide within 100 using a range of strategies	M__.6 Extend and refine efficient strategies for multiplication & division
Fractions F	F__.0 Whole number foundations; introduce manipulative	F__.1 Emerging partitioning (e.g. partitioning to create halves, thirds, etc.)	F__.2 Facile partitioning (e.g. verifying a shape has been partitioned into fourths or eighths)	F__.3 Beginning to understand a fraction as a measure, i.e. interpret $\frac{3}{4}$ as the size of 3 one-fourth pieces.	F__.4 Understand a fraction as a measure	F__.5 Comparing fractions	F__.6 Extend and refine fraction understandings

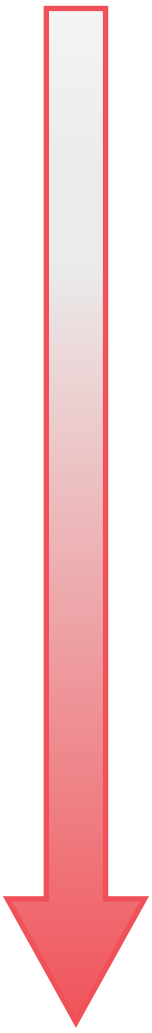
*Indicated strands align to Add+Vantage MR® (AVMR) Contructs and Levels.

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Using Numeracy Progressions

KNP activities are organized into Task Groups, a set of 4-7 related activities aligned to a numeracy progression. All activities within a task group share the same 4 digit number task group ID number. The prefix indicates the instructional strand, and the suffix, indicates the level of the activity. Within a strand, all activities with the same level will also be the same color and comparable in complexity. The example below shows the progression of one task group in the Structuring Strand.

Activity ID #	Color	Activity Name	Mathematical Task
S 2211.0	Yellow	Pyramid (Match to 5)	Students match cards representing the same amount within 5. Cards available include dot patterns, 5 frames, finger patterns, numeral, and word cards.
S 2211.1	Red	Pyramid (Make 5)	Students match two cards with a sum of 5. Cards available include dot patterns, 5 frames, finger patterns, numeral, and word cards.
S 2211.2	Blue	Pyramid (Make 10, 10 frames)	Students match two cards with a sum of 10. Cards available include both a numeral and ten frame representation.
S 2211.3	Green	Pyramid (Make 10, numeral cards)	Students match two numeral cards with a sum of 10.
S 2211.4	Purple	Pyramid (Make 20, double 10 frames)	Students match two cards with a sum of 20. Cards available include both a numeral and double ten frame representation.
S 2211.5	Pink	Pyramid (Make 20, numeral cards)	Students match two numeral cards with a sum of 20.



As the level number increases, activities increase in complexity.

In this example, note the increasing number range and choice of materials.

FNWS - Forward Number Word Sequences

Level 0	Level 1 and 2	Level 3		Level 4	
Needs to work on counting sequence 1-10 (*start-stop counting sequences)	Needs to work on "Number Word After" in range 1-10	Needs to work on counting sequence 1-30 (*start-stop counting sequences)	Needs to work on "Number Word After" in range 1-30	Needs to work on counting sequence 1-100 (*start-stop counting sequences)	Needs to work on "Number Word After" in range 1-100
KNP # → Nf__.0 and Nf__.1	KNP # → Nf__.2 and Nf__.3	KNP # → Nf__.4		KNP # → Nf__.5	

BNWS - Backward Number Word Sequences

Level 0	Level 1 and 2	Level 3		Level 4	
Needs to work on counting sequence 10-1 (*start-stop counting sequences)	Needs to work on "Number Word Before" in range 10-1	Needs to work on counting sequence 30-1 (*start-stop counting sequences)	Needs to work on "Number Word Before" in range 30-1	Needs to work on counting sequence 100-1 (*start-stop counting sequences)	Needs to work on "Number Word Before" in range 100-1
KNP # → Nb__.0 and Nb__.1	KNP # → Nb__.2 and Nb__.3	KNP # → Nb__.4		KNP # → Nb__.5	

Created by Becky Reister

Kentucky Center for Mathematics
Last Updated 7/6/2023

NID - Numeral Identification

Level 0	Level 1	Level 2	Level 3	Level 4
Needs to work on identifying numerals 0-10	Needs to work on identifying numerals 0-20	Needs to work on identifying numerals 0-100	Needs to work on identifying numerals 0-1000	Needs to work on identifying numerals 0-1,000,000
KNP # → Nf__.0 and Nf__.1	KNP # → Ni__.2	KNP # → Ni__.3	KNP # → Ni__.4	KNP # → Ni__.5

Created by Becky Reister

Kentucky Center for Mathematics
Last Updated 7/6/2023

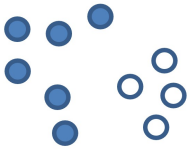
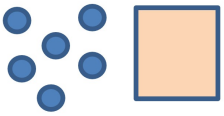
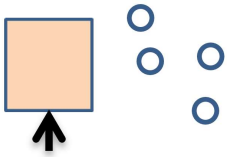
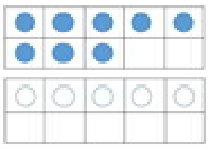
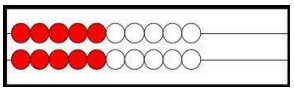
Structuring Numbers

Level 0	Level 1	Level 2	Level 3	Level 4
Needs to work on: •Subitizing regular spatial patterns to 6 •Subitizing irregular spatial patterns to 5 •Building finger patterns 1-5 simultaneously •Combining numbers in range 1-5 without counting •Partitioning numbers in range 1-5 without counting **USE MATERIALS INITIALLY, THEN TRANSITION TO BARE NUMBERS TO FIVE**	Needs to work on: •Subitizing regular spatial patterns in range 6-10 •Subitizing quantities 1-10 on ten frames •Creating simultaneous finger patterns in range of 6-10 •Combining AND partitioning numbers in range 6-10 in the context of a relevant setting (ten frame, arithmetic rack, etc.), without counting **USE MATERIALS** **REMEMBER TO WORK ON BOTH FIVE-WISE AND PAIR-WISE STRUCTURES WITHIN THESE TASKS**	Needs to work on applying knowledge of structures of five-wise and pair-wise (five-plus and doubles) to: •combine numbers in range 1-10 without counting •partition numbers in range 1-10 without counting **WORK AT THIS LEVEL CULMINATES IN BARE-NUMBER TASKS (WITHOUT SUPPORTING MATERIALS)**	Needs to work on applying knowledge of ten-plus and doubles structures to: •combine AND partition numbers in range 1-20 in the context of a relevant setting (double ten frame, arithmetic rack, etc.), without counting **USE MATERIALS** **REMEMBER TO WORK ON:** **•SUB-BASE OF FIVE** **•PAIR-WISE STRUCTURES** **•TEN-PLUS ASPECT OF TEENS**	Needs to work on applying ALL aspects of structuring (structures of five, structures of ten, doubles, structures of ten-plus) to: •combine AND partition numbers in range 1-20 without counting **WORK AT THIS LEVEL CULMINATES IN BARE-NUMBER TASKS (WITHOUT SUPPORTING MATERIALS)**
KNP # → S__.0 and S__.1	KNP # → S__.2	KNP # → S__.3	KNP # → S__.4	KNP # → S__.5

Created by Becky Reister

Kentucky Center for Mathematics
Last Updated 7/6/2023

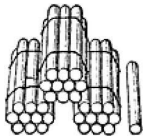
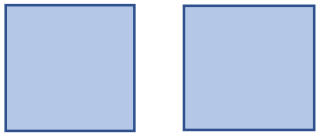
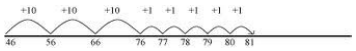
Arithmetical Strategies - Addition and Subtraction

Unitary Thinkers			Composite Thinkers	
Construct 0	Contruct 1	Construct 2	Construct 3	Construct 4
Needs to work on: •counting a collection of the same items •counting a collection of two different items	Needs to work on: •recalling how many are in a group after seeing the items, then screening •"continuing the count", by recalling how many items were screened	Needs to work on: •keeping track of counts •beginning with quantity and monitoring counts, either forward or backward (+/-) •connecting work with materials to symbolic (slowly introducing the symbolic numerals under the screens to replace the counters)	Needs to work on: •exploring different ways to solve subtractive tasks •becoming aware of strategies that are most efficient, depending on the numbers used	Needs to work on connecting structuring knowledge to additive and subtractive tasks (employing a range of non-count-by-one strategies).
 "I have some blue counters and some white counters. How many do I have altogether?"	 "I have some blues, and four whites under the cover (show quickly). How many do I have altogether?" *begin with 2-3 hidden in regular dot pattern *next use 2-3 in random order *next use 4-6 in regular dot pattern *follow this with 4-6 in random order	 (any range of numbers with which the student is facile) "I have ___ counters here (show quickly), and four more. How many altogether?" Move to covering both addends and covered subtraction tasks.	  Use of settings such as these support movement from Construct 3 to Construct 5. Connect to Structuring Strand. Activities could include flashing the image and asking questions such as: *How many did you see? *How many more would make 20? *What if I added two? *What if I took three away? * How did you figure that out? Also see KNP # S__.4 and S__.5	
KNP # → A__.0 and A__.1	KNP # → A__.2	KNP # → A__.3	KNP # → A__.4	KNP # → A__.5

Created by Becky Reister

Kentucky Center for Mathematics
Last Updated 7/6/2023

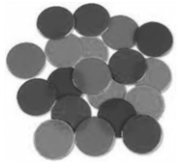
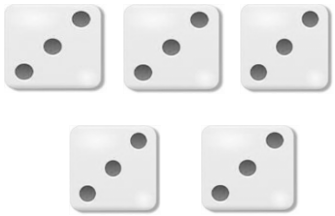
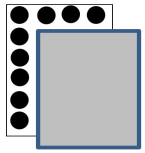
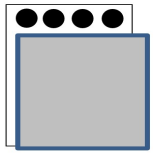
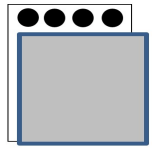
Conceptual Place Value (Base Ten Reasoning)

Construct 0	Construct 1	Construct 2	Construct 3	Construct 4-5
Needs to work on grouping items into 10s; counting by 10s on the decade in the context of materials	Needs to work on counting by 10s on and off the decade in the context of materials (incrementing/ decrementing)	Needs to work on solving 2 digit addition and subtraction tasks with the gradual removal of materials; record mental strategies with notation	Needs to work on using varied mental strategies for solving 2 digit addition and subtraction tasks without materials; i.e. "can you solve in a different way?"	Needs to add and subtract 3 digit numbers using a range of mental strategies
  "30, 40, 50, 60,…"	 (a bundle is shown, then placed under the screen as counting continues) "31, 41, 51, 61, …"	 	JUMP SPLIT-JUMP SPLIT OVER-JUMP TRANSFORM	Put all of the skills together!
KNP # → T__.0 and T__.1	KNP # → T__.2	KNP # → T__.3	KNP # → T__.4	KNP # → T__.5

Created by Becky Reister

Kentucky Center for Mathematics
Last Updated 7/6/2023

Multiplicatin and Division

Construct 0	Construct 1	Construct 2	Construct 3	Construct 4-5
Needs to work on putting items in equal groups; sharing items equally	Needs to work on counting visible, pre-grouped items and associate stress and skip counting with quantities	Needs to work on counting groups of items where individual items are not visible	Needs to work on counting groups within a group of non-visible items; different ways to break a whole group down without perceptual markers	Needs to work on recall or quick computation of basic mult/div facts; work on recognizing inverse relationship of mult/div and commutativity of mult
← **Work on stress or skip counting throughout** →				
"You have 20 cookies. If you shared them equally among five people, how many cookies would each person get?" 	"How many dots all together?" 	If you know that there are six rows of four dots, how many dots are there all together? 	"If you know that there are six rows of four dots, how many dots are there all together?" 	"How many rows like this would you need to uncover to show 8 dots? What about 16? What about 32?" 
KNP # → M__.0 and M__.1	KNP # → M__.2	KNP # → M__.3	KNP # → M__.4	KNP # → M__.5 and M__.6

Created by Becky Reister

Kentucky Center for Mathematics
Last Updated 7/6/2023

Forward Counting

- Yellow: I am learning to count forward to 10, starting at any number.
- Red: I am learning to count forward to 30, starting at any number.
- Blue: I am learning to count forward to 100, starting at any number.
- Green: I am learning to count forward to 1000, starting at any number.

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Backward Counting

- Yellow: I am learning to count backward, starting at any number in the range 1 to 10.
- Red: I am learning to count backward, starting at any number in the range 1 to 30.
- Blue: I am learning to count backward, starting at any number in the range 1 to 100.
- Green: I am learning to count backward, starting at any number in the range 1 to 1000.

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Numeral Identification (NID)

- **Yellow: I am learning to read numerals 1 to 10.**
- **Red: I am learning to read numerals 1 to 20.**
- **Blue: I am learning to read numerals 1 to 100.**
- **Green: I am learning to read numerals 1 to 1000.**
- **Purple: I am learning to read numerals 1 to 1,000,000.**

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Addition and Subtraction

- Yellow: I am learning to count items I can see.
- Red: I am learning to add by counting two groups of items that I can see.
- Blue: I am learning to add items I can't see by counting from 1.
- Green: I am learning to count on to add. I am learning to count back to subtract.
- Purple, Pink & Orange: I am learning to use different strategies to add and subtract.

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Structuring Numbers

Yellow: I am learning to subitize amounts up to 6.

Red: Facile to 5

- **I am learning to immediately show amounts up to 5 on my fingers.**
- **I am learning to easily (fluently) add and subtract within 5.**

Blue: Intermediate to 10

- **I am learning to quickly recognize amounts up to 10 (shown pair-wise or five-wise).**
- **I am learning to immediately show amounts up to 10 on my fingers.**
- **I am learning to add and subtract within 10 with support of materials such as dot card or fingers.**

Green: Facile to 10

- **I am learning to easily say numbers that add together to make any number up to 10.**
- **I am learning to easily (fluently) add and subtract within 10.**

Purple: Intermediate to 20

- **I am learning to easily add doubles and near doubles to 20**
- **I am learning to easily add 10 to any number 1-10. I am learning to easily separate the numbers 11 to 20 into 10 and some more.**
- **I am learning to add within 20 with support of materials such as ten frames or a rekenrek.**

Pink: Facile to 20

- **I am learning to easily (fluently) add and subtract within 20.**

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Place Value (Tens and Ones)

- **Yellow: I am learning to add or subtract (ones or groups of ten only) with materials within 100**
- **Red: I am learning to add or subtract within 100 with materials**
- **Blue: I am learning to add or subtract within 100 without materials**
- **Green: I am learning to add or subtract within 100 using lots of different strategies**
- **Purple: I am learning to add or subtract within 1000 using lots of different strategies**

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources

Multiplication and Division

- Red: I am learning to share items into equal groups. I am learning to make equal groups. I am learning to describe items arranged into equal groups.
- Blue: I am learning to count items arranged into groups using stress or skip counting.
- Green: I am learning to figure out "how many" if items are arranged into groups and I can see the groups and not the items in the group.
- Purple: I am learning to multiply and divide. I usually count to solve.
- Pink and Orange: I am learning to efficiently multiply and divide. I use known facts, the inverse relationship between multiplication and division, and other strategies.

KCM
KENTUCKY CENTER FOR MATHEMATICS
Kentucky Numeracy Project
Instructional Resources